# Accelerated Mathematics 

## CHAPTER 9

## GEOMETRIC PROPERTIES

## PART I

## Topics Covered:

- Geometry vocabulary
- Similarity and congruence
- Classifying quadrilaterals
- Transformations (translations, reflections, rotations, dilations)


LIFE IS POINTLESS

HE TOOK THE RHOMBUS.


Geometry is the area of mathematics that deals with the properties of points, lines, surfaces, and solids. It is derived from the Greek "geometra" which literally means earth measurement.

## GEOMETRIC PSYCHOLOGY



SQUARE - hard worker, likes structured and organized environment, loves data, dependable, tenacious, likes to do the job themselves, likes things in writing, makes sure things get done well, likes lots of details, will not tolerate sloppy work


TRIANGLE - leader, very focused, loves recognition, very sure people, outspoken, very focused on goal at hand, loves lists and sticky notes, independent, likes to do his/her own thing, always get the best deals


RECTANGLE - sick of being a square and reaching upward like a triangle, excited, unpredictable, excellent student, less frozen than other students, team players, thinks well in groups


CIRCLE - likes harmony, fun, nurturing, caretaker, loves people with problems so that they can help them solve problems, best listener and best communicator, has good gut ideas, trustworthy, cannot stand conflict, , have a hard time saying no, has many friends


SCRIBBLE - open-ended, most creative, highly conceptual broad ideas, asks "what if" a lot, future oriented, not a detailed person, has lots of ideas both good and bad, good trouble shooter, has a short attention span



Acute Triangle



Trapezium (Amer. Eng.)


Trapezoid (Amer. Eng.) Trapezium (Brit. Eng.)


Isosceles trapezoid (Am.) Isosceles trapezium (Br.)


Kite


Rhombus


Rectangle


Parallelogram


Square


## Properties of Similar and Congruent Shapes

http://www.virtualnerd.com/geometry/similarity/polygons/similar-figures-missing-measurement-example
http://www.virtualnerd.com/geometry/similarity/triangles/indirect-measurement-example
http://www.purplemath.com/modules/ratio6.htm
http://www.mathwarehouse.com/geometry/similar/triangles/sides-and-angles-of-similar-triangles.php

Similar figures are the same shape but may be different sizes.

## $\triangle A B C \sim \triangle D E F$

Corresponding angles are congruent.

$$
\angle A \cong \angle D \quad \angle B \cong \angle E \quad \angle C \cong \angle F
$$

Corresponding side lengths are proportional.


## Congruent figures have the exact same shape and size.

## $\triangle A B C \cong \triangle D E F$

## Corresponding angles are congruent.

$\angle A \cong \angle D$
$\angle B \cong \angle E$
$\angle C \cong \angle F$


Corresponding side lengths are congruent.
$\overline{A B} \cong \overline{D E}$
$\overline{A C} \cong \overline{D F}$
$\overline{B C} \cong \overline{E F}$

Is there enough information to prove whether these triangles are similar? If so, are they?


Set up a proportion to tell whether each pair of polygons is similar.
1.



3 cm

3.

4.

5.


For each pair of similar figures write a proportion and use the proportion to find the length of $x$. Use a separate sheet of paper.
6.

8.
 30 cm

7.

9.


For each pair of similar figures write a proportion and use the proportion to find the length of $x$. Use a separate sheet of paper.
5.

## Tell whether the shapes below are similar. Explain your answer.

1. 



2.

3.


4.


Solve

| 5. | A rectangle made of square tiles measures 8 tiles wide and 10 tiles long. What <br> is the length in tiles of a similar rectangle 12 tiles wide? |  |
| :--- | :--- | :--- |
| 6. | A computer monitor is a rectangle. Display A is 240 pixels by 160 pixels. <br> Display B is 320 pixels by 200 pixels. Is Display A similar to Display B? <br> Explain. |  |

## The figures in each pair are similar. Find the unknown measures.

7. 


9.


8.


Janelle drew $\overline{K L}$ in isosceles trapezoid FGHJ to create similar trapezoids FKLJ and KGHL.


Based on the given information, what are the values of $y$ and $w$ in centimeters?
Pentagon $A B C D E$ is similar to pentagon RSTUV. The perimeter of pentagon $A B C D E$ is 36.8 centimeters.


What is the perimeter of pentagon RSTUV?

## AMBIGRAMS

A graphic artist named John Langdon began to experiment in the 1970s with a special way to write words as ambigrams. Look at all the examples below and see if you can determine what an ambigram is.


## GEOMETRIC TRANSFORMATIONS

Translation - Slide
Reflection - Flip
Rotation - Turn
Dilation - Enlargement or Reduction
Preimage - the shape before it is transformed Image - the shape after it is transformed

| Translation | $(x, y) \rightarrow(x+a, y+b)$ |
| :---: | :---: |


| Reflection over the $y$-axis | $(x, y) \rightarrow(-x, y)$ |
| :--- | :--- |
| Reflection over the $x$-axis | $(x, y) \rightarrow(x,-y)$ |


| All rotations below are centered about the origin. |  |  |
| :---: | :---: | :---: |
| Rotation $90^{\circ}$ clockwise | $(x, y) \rightarrow(y,-x)$ | Rotation $270^{\circ}$ counterclockwise |
| Rotation $180^{\circ}$ clockwise | $(x, y) \rightarrow(-x,-y)$ | Rotation $180^{\circ}$ counterclockwise |
| Rotation $270^{\circ}$ clockwise | $(x, y) \rightarrow(-y, x)$ | Rotation $90^{\circ}$ counterclockwise |


| All dilations below are centered about the origin. |  |
| :---: | :---: |
|  | $(x, y) \rightarrow(k x, k y)$ |
| Dilation | $k$ is called the scale factor |
| $k<1$ means reduction |  |
| $k>1$ means enlargement |  |


|  | Translation | Rotation | Reflection | Dilation |
| :---: | :---: | :---: | :---: | :---: |
| Changes Orientation |  | $V$ | $\checkmark$ |  |
| Changes Location | $V$ | $V$ | $\checkmark$ | $\checkmark$ |
| Changes Size |  |  |  | $\checkmark$ |

Consider the triangle shown on the coordinate plane.


1. Record the coordinates of the vertices of the triangle.
2. Translate the triangle down 2 units and right 5 units. Graph the translation.
3. A symbolic representation for the translated triangle would be: $(x, y) \rightarrow(x+5, y-2)$


| 4. | Write a verbal description of the translation. |  |
| :--- | :--- | :--- | :--- |
| 5. | Describe the translation above using symbolic representation. |  |



Determine the coordinates of the vertices for each image of trapezoid STUW after each of the following translations in performed.

| 1. | 3 units to the left and 3 units down |  |
| :---: | :---: | :--- |
| 2. | $(x, y) \rightarrow(x, y-4)$ |  |
| 3. | $(x, y) \rightarrow(x-2, y+1)$ |  |
| 4. | $(x, y) \rightarrow(x-4, y)$ |  |
| 5. | Find a single transformation that has the same effect <br> as the composition of translations <br> $(x, y) \rightarrow(x-2, y+1)$ followed by <br> $(x, y) \rightarrow(x+1, y+3)$. |  |


| 6. | Draw triangle ABC at points $(-1,6),(-4,1),(1,3)$. |  |
| :---: | :--- | :--- |
| 7. | Draw triangle $\mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{C}^{\prime}$ is located at $(5,2),(2,-3),(7,-1)$ |  |
| 8. | Write a symbolic representation for the translated triangle compared to the original. |  |
| 9. | Write a description (in words) of this translation. |  |
| 10. | Connie translated trapezoid $R S T U$ to trapezoid <br> $R^{\prime} S^{\prime} T^{\prime} U^{\prime}$. Vertex S was at $(-5,-7)$. If vertex $S^{\prime}$ is at <br> $(-8,5)$, write a description of this translation. | Move each vertex __units to the <br> and units |

Describe the translation that maps point $\boldsymbol{A}$ to point $\boldsymbol{A}^{\prime}$.
1.

2.


## Draw the image of the figure after each translation.

3. 3 units left and 9 units down

4. 3 units right and 6 units up

5. a. Graph rectangle $J^{\prime} K^{\prime} L^{\prime} M^{\prime}$, the image of rectangle $J K L M$, after a translation of 1 unit right and 6 units up.
b. Find the area of each rectangle.
c. Is it possible for the area of a figure to change after it is translated? Explain.
$\qquad$
$\qquad$


## Use triangles $A B C$ and $A^{\prime} B^{\prime} C^{\prime}$ for Exercises 1-4.

1. Use the coordinates to find the lengths of the sides.

Triangle : $=\ldots \quad=$
Triangle : = $\qquad$ ; $=$ $\qquad$
2. Find the ratios of the corresponding sides.
$\qquad$
$-=$
$\square=-$ $\qquad$
3. Is triangle
a dilation of triangle $\qquad$

4. If triangle is a dilation of triangle , is it a reduction or an enlargement? $\qquad$

For Exercises 5-8, tell whether one figure is a dilation of the other or not. If one figure is a dilation of the other, tell whether it is an enlargement or a reduction. Explain your reasoning.

| Triangle has sides of $3 \mathrm{~cm}, 4 \mathrm{~cm}$, and 5 cm . Triangle has sides of $12 \mathrm{~cm}, 16 \mathrm{~cm}$, and 25 cm . |  |
| :---: | :---: |
| Quadrilateral has coordinates of $\quad(0,0),(0,4),(-6,4)$, and$(-6,0)$. Quadrilateral has coordinates of $(0,0),(0,2)$, <br> $(-3,2)$, and $(-3,0)$.  |  |
| Triangle has sides of $4 \mathrm{~cm}, 4 \mathrm{~cm}$, and 7 cm . Triangle has sides of $12 \mathrm{~cm}, 12 \mathrm{~cm}$, and 21 cm . |  |
| Does the following figure show a dilation? Explain. |  |

Draw a dilation of the figure using the given scale factor, $k$.

1. $k=2$

2. $k=\frac{1}{2}$

3. $k=\frac{1}{4}$

4. $k=1 \frac{1}{2}$


Determine whether the dilation from Figure A to Figure B is a reduction or an enlargement. Then use a proportion to find the values of the variables.
5.

6.

7.

8.


For each movement below complete the table.

|  |  | Type of transformation | Does it <br> maintain <br> congruence? | Does it <br> maintain <br> orientation? |
| :--- | :--- | :--- | :--- | :--- |
| 1. | $(x, y) \rightarrow(x-15, y+3)$ |  |  |  |
| 2. | A flip across the y-axis |  |  |  |
| 3. | $(x, y) \rightarrow(x,-y)$ |  |  |  |
| 4. | A turn of 180 degrees |  |  |  |
| 5. | $(x, y) \rightarrow(2 x, 2 y)$ |  |  |  |
| 6. | A slide 3 units to the right |  |  |  |
| 7. | $(x, y) \rightarrow\left(\frac{1}{6} x, \frac{1}{6} y\right)$ |  |  |  |
| 8. | An enlargement with the area <br> 9 times larger |  |  |  |
| 9. | $(x, y) \rightarrow(y,-x)$ |  |  |  |
| 10. | A turn of 90 degrees <br> clockwise |  |  |  |
| 11. | $(x, y) \rightarrow(-x,-y)$ |  |  |  |
| 12. | $(x, y) \rightarrow(-y, x)$ |  |  |  |

If the listed transformation is a dilation, complete the row. If not, leave the row blank.

|  |  | Pre-Image Area | Enlargement or <br> Reduction | Image Area |
| :---: | :--- | :---: | :---: | :---: |
| 13. | $(x, y) \rightarrow(2 x, 2 y)$ | 40 sq. units |  |  |
| 14. | $(x, y) \rightarrow\left(\frac{1}{2} x, \frac{1}{3} y\right)$ | 24 sq. units |  |  |
| 15. | $k=2.5$ | 100 sq. units |  | 288 sq. units |
| 16. | $(x, y) \rightarrow\left(\frac{1}{6} x, \frac{1}{6} y\right)$ |  |  | $\frac{25}{}$ sq. units |
| 17. | $(x, y) \rightarrow(y, x)$ |  |  | sq. units |
| 18. | $k=\frac{3}{2}$ |  |  |  |
| 19. | $(x, y) \rightarrow(10 x, 10 y)$ |  |  | $c$ sq. units |
| 20. | $(x, y) \rightarrow(k y, k x)$ |  |  |  |

This project is designed to conclude geometric transformations with students. It includes a review of translations, reflections and rotations on the coordinate grid. Each student has a unique product to make, but they can help each other as needed.

## Creating Your Own Emblem

You are going to use your name, a coordinate graph, and some transformations to find your unique emblem.
$\square$ First, in the name chart, write the first 6 letters of your first name. If your name is less than 6 letters long, start over on your name.
$\square$ Now write in the first 6 letters of your last name. Again if you need more letters start over at the beginning of your name.
$\square$ Use the letter-to-number conversion chart to get the coordinates for your original shape. The X coordinate comes from the first name and the Y coordinate comes from the last name. If any ordered pairs duplicate, switch the x - and y -coordinates so that all coordinate pairs are unique.
$\square$ Graph your original shape on the coordinate grid on the next page by connecting the points along the perimeter of the shape. Make sure your points are connected to form a closed figure. (This may mean that points are not be connected in order.)

| Letter | Value | Letter | Value | Letter | Value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| AB | 10 | IJ | 7 | QR | 4 |
| CD | 2 | KL | 6 | ST | 14 |
| EF | 3 | MN | 5 | UV | 11 |
| GH | 8 | OP | 9 | WXYZ | 1 |

Original Figure

$\boldsymbol{x} \quad \boldsymbol{y} \quad$ Coordinate

A

|  |  |  |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

## Project Directions

## Graph your Original Figure

$\square$ Using the coordinates from the previous page, graph your original figure. Be sure to rewrite your coordinates in the table provided.
$\square$ Make sure ALL shapes you graph form closed figures.

## Translation $(x-5, y-8)$

A translation is taking the original image and sliding it without turning it.
Graph your original shape again.
$\square$ Now translate the shape. Find the coordinates for the image. Graph the image.

## Reflection in the $x$-Axis

A reflection is taking the original image and flipping it along a line of reflection.
Graph your original shape again.
$\square$ Now reflect the shape over the x-axis. Find the coordinates for the image. Graph the image.

## $90^{\circ}$ Clockwise Rotation about the origin

$\square$ Graph your original shape again.
$\square$ Rotate the figure $\mathbf{9 0}$ degrees clockwise. Find the coordinates for the image. Graph the image.

## $180^{\circ}$ Rotation about the origin

$\square$ Graph your original shape again.
$\square$ Rotate the figure $\mathbf{1 8 0}$ degrees. Find the coordinates for the image. Graph the image.

## Your Emblem

Now to make your emblem, which will stand for you:
Graph your original shape.
Perform a sequence of two unique transformations on your original image.
$\square$ Your emblem will consist of THREE figures:
$\square$ Figure 1: Your original shape (pre-image)
$\square$ Figure 2: Your original shape transformed using a translation (image)
Figure 3: The image transformed using a reflection or rotation
$\square$ Clearly state the sequence of transformations that you used in your emblem.
$\square$ Color or decorate. Think of a slogan or motto to go with your emblem.

Original Figure


|  | Original Figure |  |
| :--- | :--- | :--- |
|  | $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| A |  |  |
| B |  |  |
| C |  |  |
| D |  |  |
| E |  |  |
| F |  |  |
|  |  |  |
|  |  |  |

Transformation \#1: $\quad$ Translation $(x-5, y-8)$


Transformation \#2: $\quad$ Reflection across the $\boldsymbol{x}$-Axis


Transformation \#3: $\quad 90^{\mathbf{0}}$ Clockwise Rotation



## Emblem Name:

$\qquad$


1. $\qquad$
2. $\qquad$
MY MOTTO

## Accelerated Mathematics

## Chapter 11

## DIMENSIONAL GEOMETRY

## Topics Covered:

- Naming 3D shapes
- Nets
- Volume of Prisms
- Volume of Pyramids
- Surface Area of Prisms
- Surface Area of Pyramids
- Surface Area using Nets
- Volume of Cylinders
- Volume of Cones
- Volume of Spheres
- Surface Area of Prisms
- Surface Area of Cylinders



## Linear Equations

| Slope-intercept form | $y=m x+b$ | Direct Variation | $y=k x \quad\left(8^{\text {th }} \mathrm{gr}\right.$ |
| :--- | :--- | :--- | :--- |
| Constant of proportionality | $k=\frac{y}{x}$ | Slope of a line | $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$ |
| Circumference | Circle | $C=2 \pi r$ or $C=\pi d$ |  |
| Area | Trapezoid | $A=\frac{1}{2}\left(b_{1}+b_{2}\right) h$ |  |
| $\quad$ Rectangle | $A=b h$ | $A=\pi r^{2}$ |  |
| Parallelogram | $A=b h$ |  |  |
| Triangle | $A=\frac{b h}{2}$ or $A=\frac{1}{2} b h$ |  |  |


| Surface Area ( $8^{\text {th }}$ grade) |  | Lateral | Total |
| :---: | :---: | :---: | :---: |
| Prism |  | $S=P h$ | $S=P h+2 B$ |
| Cylinder |  | $S=2 \pi r h$ | $S=2 \pi r h+2 \pi r^{2}$ |
| Volume |  |  |  |
| Triangular prism | $V=B h$ | Cylinder grade) | $V=B h$ or $V=\pi r^{2} h \quad\left(8^{\text {th }}\right.$ |
| Rectangular prism | $V=B h$ | Cone | $V=\frac{1}{3} B h \text { or } V=\frac{1}{3} \pi r^{2} h \quad\left(8^{\text {th }}\right)$ |
| Pyramid | $V=\frac{1}{3} B h$ | Sphere | $V=\frac{4}{3} \pi r^{3} \quad\left(8^{\text {th }} \text { grade }\right)$ |

Pi

$$
\pi \approx 3.14 \text { or } \pi \approx \frac{22}{7}
$$

| Distance | $d=r t$ | Compound Interest | $A=P(1+r)^{t}$ |
| :--- | :--- | :--- | :--- |
| Simple Interest | $I=p r t$ | Pythagorean Theorem | $a^{2}+b^{2}=c^{2} \quad\left(8^{\text {th }}\right.$ grade $)$ |


| Customary - Length | Customary - Volume/Capacity | Customary - Mass/Weight |
| :---: | :---: | :---: |
| 1 mile $=1760$ yards | 1 pint $=2$ cups | 1 ton $=2,000$ pounds |
| 1 yard $=3$ feet | 1 cup $=8$ fluid ounces | 1 pound $=16$ ounces |
| 1 foot $=12$ inches | 1 quart $=2$ pints |  |
|  | 1 gallon $=4$ quarts |  |
| Metric - Length | Metric - Volume/Capacity | Metric - Mass/Weight |
| 1 kilometer $=1000$ meters | 1 liter $=1000$ milliliters | 1 kilogram $=1000$ grams |
| 1 meter $=100$ centimeters |  | 1 gram $=1000$ milligrams |
| 1 centimeter $=10$ millimeters |  |  |

## EXAMPLES

$$
\begin{aligned}
& A=\frac{1}{2}\left(b_{1}+b_{2}\right) h \\
& A=\frac{1}{2}(10+20) \cdot 6 \\
& A=90 \mathrm{~cm}^{2} \\
& V=\pi r^{2} h \\
& S=2 B+P h \\
& V=3.14 \bullet 10^{2} \bullet 5 \quad S=2(8 \bullet 6)+(28) \bullet 10 \\
& V=1570 \mathrm{~m}^{3} \\
& S=376 \text { in }^{2}
\end{aligned}
$$

Cube


Lateral face: A face that joins the
bases of a solid. It is any edge or face that is not part of the base.

base



Cylinder


Neither cylinders nor cones have edges.


## Three - dimensional (solid) figures include what five shapes?

|  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |

## Characteristics of Solids

- Three-dimensional figures, or solids, can have $\qquad$ or $\qquad$ surfaces.
- Prisms and pyramids are named by the shapes of their $\qquad$ .

- A $\qquad$ is a diagram of the surfaces of a three-dimensional figure. It can be folded to form the three-dimensional figure.


Classify each solid and tell how many faces, edges, and vertices.

| Type | Properties |  |  |
| :--- | :--- | :--- | :--- |
| 1. | Faces $=$ | Edges $=$ | Vertices $=$ |
| 2. | Faces $=$ | Edges $=$ | Vertices $=$ |
| 3. | Faces $=$ | Edges $=$ | Vertices $=$ |
| 4. | Faces $=$ | Edges $=$ | Vertices $=$ |
| 5. | Faces $=$ | Edges $=$ | Vertices $=$ |


| $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |

The volume of a solid is a measure of the amount of space it occupies or how much it can hold.
The volume $V$ of a prism is the product of the area of the base $B$ and the height $h . V=B h$
The volume $V$ of a pyramid is one third the product of the area of the base, $B$ and the height, $h$.
$V=\frac{1}{3} B h$


$$
\begin{array}{c|c}
\hline \text { Volume of a Prism } & \text { Volume of a Pyramid } \\
V=B h, B=\text { Area of the base } & V=\frac{1}{3} B h, B=\text { Area of the base } \\
\hline
\end{array}
$$

## Find the volume of each figure.

1. 


2.


Identify the three-dimensional shape that can be formed from each net.
3.

4.

5.


Solve.

| 6. | The base of a rectangular pyramid is 13 inches long and 12 inches wide. The <br> height of the pyramid is 8 inches. What is the volume of the pyramid? |  |
| :---: | :--- | :--- |
| 7. | A cake pan is shaped like a rectangular prism. The pan's volume is 216 in $^{3}$. The <br> cake pan has a base that is 12 inches by 9 inches. What is the height of the cake <br> pan? |  |
| 8. | A form for a garden ornament is made up of two shapes, a cube and a square <br> pyramid (see picture at the right above this table). To make an ornament the <br> form is filled with concrete. What is the volume of the form? |  |

Find the volume of the four prisms below.

2.

3.

4.


Find the volume of the rectangular prism with length, $l$, width, $w$, and height, $h$.
5. $l=5 \mathrm{~m}, w=8 \mathrm{~m}, h=9 \mathrm{~m}$
6. $l=10 \mathrm{in}, w=14 \mathrm{in}, h=15 \mathrm{in}$
7. $l=16 \mathrm{yd}, w=10.2 \mathrm{yd}, h=4.3 \mathrm{yd}$
8. $l=12 \mathrm{~mm}, w=17 \mathrm{~mm}, h=2 \frac{1}{2} \mathrm{~mm}$

Find the volume of the solid. Round your answer to the nearest hundredth, if necessary.
9.

10.

29 m

Find the volume of each pyramid below.



This is the Pyramid Arena in Memphis, TN.

Find the volume of the pyramid.
3. Triangular pyramid: base of the triangle $=8 \mathrm{ft}$, height of the triangle $=6 \mathrm{ft}$, height of the pyramid $=7$ ft
4. Square pyramid: sides of the square $=14 \mathrm{~mm}$, height of the pyramid $=9 \mathrm{~mm}$
5. A pyramid bookend is being formed out of concrete. The rectangular base on the bookend is 8 in by 7 in . The height of the pyramid is 5 in .

Find the volume of the pyramid with base area $B$ and height $h$.
6. $B=18 \mathrm{in}^{2}, h=5 \mathrm{in}$
7. $B=6.3 \mathrm{~mm}^{2}, h=2.9 \mathrm{~mm}$

Find the volume of the three pyramids below.


Find the volume of the square pyramid with base side length $s$ and height $h$.
11. $s=3$ in, $h=7$ in
12. $s=9 \mathrm{~mm}, h=14 \mathrm{~mm}$
13. $s=9 \mathrm{ft}, h=\frac{1}{2} \mathrm{ft}$

Find the volume of each solid below.
1.

2.

3.

4.


Solve.

| 5. | A triangular prism has a base area of 20 square feet and a height of 4 feet. Find the volume. |
| :---: | :---: |
| 6. | The volume of a triangular pyramid is 300 cubic meters. What is the area of the pyramid's base if the pyramid height is 3 meters? |
| 7. | A triangular prism has a volume of 2,500 cubic feet. What is the length of the prism if its triangular bases are right triangles, each with perpendicular sides of 10 and 20 feet? |
| 8. | The height of a pyramid is 15 inches. The pyramid's base is a square with a side of 5 inches. What is the pyramid's volume? |
| 9. | A rectangular box has a volume of 480 cubic inches. The height of the box is 5 inches. The ratio of the length of the box to the width of the box is 3 to 2 . What is the measure of the width of the box? |
| 10. | A square pyramid has edges of length $p$ and a height of $p$ as well. Which expression represents the volume of the pyramid? <br> A $\frac{1}{3} p^{3}$ <br> B $\frac{1}{3} p^{2}$ <br> C $\frac{1}{3} p^{2}+p$ <br> D $\frac{1}{3} p+p^{2}$ |
| 11. | Mr. Underwood says that when the height of a rectangular prism is doubled, its volume also doubles. Mrs. Scogin says then when the height of a rectangular prism is doubled, volume quadruples. Who is correct? Explain your reasoning. |
| 12. | What happens to the volume of a rectangular prism when its height is tripled? |
| 13. | What happens to the volume of a triangular pyramid when all dimensions are tripled? |
| 14. | What happens to the volume of a triangular prism when the area of its base is doubled? |

A net is a two-dimensional pattern that forms a solid when it is folded.
The surface area of a polyhedron is the sum of the areas of its faces.
The total surface area of a prism is the sum of twice the area of the base and the product of the base's perimeter and the height. $S=P h+2 B$

## Example

Draw a net for the pentagonal prism. For the rectangular faces, draw adjacent rectangles. Draw the bases on opposite sides one rectangle.


Draw a net for the following shapes.


A lamp shade will be constructed from rice paper shown below. How much paper will be needed to make the lampshade? The first time, use the sum of the areas method. The second time, use the formula for the surface area of a prism. The triangles are equilateral triangles.


The total surface area of a solid is the sum of the areas of all faces including the bases.
The lateral surface area of a solid is the sum of all faces excluding the bases. In a rectangular prism, you can assume the bases are the top and bottom faces, unless otherwise specified.


Top


Bottom


Front



Rectangular Prism


A 3D Rectangular Prism can be unfolded to create a flat 2D shape, called the "Net" of the Prism.

Surface Area from a Net of a Prism or Pyramid
Total Surface Area $S=$ Add the area of all sides and base(s) $S=$ Add the area of all sides

## Surface Area of a Prism

| Total Surface Area | Lateral Surface Area |
| :---: | :---: |
| $S=P h+2 B$, | $S=P h$ |
| $P=$ Perimeter of base, $B=$ Area of base | $P=$ Perimeter of base |

Please measure to the nearest tenth of a centimeter.


Dimensions: $\qquad$ , $\qquad$ , $\qquad$

LS = $\qquad$
$S=$ $\qquad$

## Partner 1: Calculate the Surface Area.



## Partner 2: Calculate the Volume.

Is that the only possible prism? Can you find another one that works? Or many more that work?
Calculate the surface area and volume for a variety of rectangular prisms. Your goal is to have the numbers end up the same so put some thought into your numbers instead of just random guesses. You can also use centimeter cubes if you like.

How are you choosing your prisms to test? Is there any kind of pattern?
Complete the chart with the prisms that you test.

| Dimension 1 | Dimension 2 | Dimension 3 | Volume | Surface Area |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |
|  |  |  |  |  |
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|  |  |  |  |  |

Find the total and lateral surface area of each figure. Assume the top and bottom faces are the bases. Don't forget to include units!
1.

3.

2.

4.

5. A
glass,
equilateral triangular prism for a telescope is 5.5 inches long. Each side of the prism's triangular bases is 4 inches long and 3 inches high. How much glass covers the surface of the prism?

Draw a net, find the area of each face, and find the total of all the areas
6.

7.


1. You are making a jewelry box shown for your mother. Find surface area TWO WAYS. First, draw a net and then find the amount of wood you need to make the box. Then, use the formula for the surface area of a prism.

2. What is the surface area of a rectangular prism that is 6 inches long, 8 inches wide, and 2 inches high?

Find the surface area of the prism, where $B$ is the area of the base, $P$ is the perimeter of the base, and $h$ is the height.
3. $B=8 \mathrm{~m}^{2}, P=3 \mathrm{~m}, h=6 \mathrm{~m} \quad$ 4. $B=15 \mathrm{~m}^{2}, P=12 \mathrm{~m}, h=3 \mathrm{~m}$
5. $B=42 \mathrm{yd}^{2}, P=23 \mathrm{yd}, h=8 \mathrm{yd}$
6. $B=58 \mathrm{~mm}^{2}, P=36 \mathrm{~mm}, h=20 \mathrm{~mm}$

Identify the solid shown by the net. Then find the surface area.
7.

8.


Draw a net for the solid. Then find the surface area of the solid.

11.

9. 10 .

12.


Created by Lance Mangham, $6^{\text {th }}$ grade math, Carroll ISD

The slant height, $l$, of a regular pyramid is the height of any of its triangular faces.
The surface area, $S$, of a regular pyramid is the sum of the area of the base $B$ and one half the product of the base perimeter $P$ and the slant height $l . \quad S=B+\frac{1}{2} P l$

What is the height of the pyramid? $\qquad$
What is the slant height of the pyramid?
What is the base shape? $\qquad$
What does B stand for? $\qquad$
What does P stand for? $\qquad$
$\qquad$
$\qquad$
LSA =
TSA $=$
$\mathrm{V}=$ $\qquad$


Find the total surface area of the four regular pyramids shown by drawing a net first. Then find the lateral surface area of each.
1.

2.

3.

4.

5. Find the surface area of a pyramid whose slant height is 9 cm and whose base is a 4 cm by 6 cm rectangle.

Find the surface area of the rectangular pyramid with the base lengths shown and the slant height, $l$.
6. length $=$ width $=4 \mathrm{~m}, l=13 \mathrm{~m}$
8. length $=$ width $=11 \mathrm{~m}, l=17.3 \mathrm{~m}$
10. The top of the Washington Monument is a triangular pyramid with a square base. Each triangular face is 58 feet tall and 34 feet wide and covered with white marble. About how many square feet of marble cover the faces of the pyramid?

1. What is the total surface area of this cube?

A $91 \frac{1}{8} \mathrm{ft}^{2}$
B $\quad 20 \frac{1}{4} \mathrm{ft}^{2}$
C $121 \frac{1}{2} \mathrm{ft}^{2}$
D $\quad 101 \frac{1}{4} \mathrm{ft}^{2}$

Find the surface area of the following shapes.
2.


Draw a net and find the area of each face to calculate the lateral and total surface areas for each figure.
4. The base is a square.

5.

6.

7. You are wrapping a gift box that is 15 inches long, 12 inches wide, and 4 inches deep. Use a net to find the length and width of a single sheet of paper that could be used to wrap the entire gift box. Find the surface area of the box.

Length of gift paper: $\qquad$ Width of gift paper: $\qquad$ Surface Area: $\qquad$
\(\left.$$
\begin{array}{|l|l|l|}\hline \text { 1. } & \begin{array}{l}\text { A shipping company sells two types of cartons that are shaped like } \\
\text { rectangular prisms. The larger carton has a volume of } 720 \text { cubic inches. } \\
\text { The smaller carton has dimensions that are half the size of the larger } \\
\text { carton. What is the volume, in cubic inches, of the smaller carton? }\end{array} & \\
\hline \text { 2. } & \begin{array}{l}\text { An ice cream carton has a volume of } 64 \text { fluid ounces. A second ice } \\
\text { cream carton has dimensions that are three-fourths the size of the larger } \\
\text { carton. What is the volume of the smaller carton? }\end{array}
$$ \& <br>

\hline 3. \& How many 2 by 2 by 2 inch cubes will fit into a 4 by 8 by 12 inch box?\end{array}\right]\)| For a regular pentagonal prism, what is the ratio of the number of vertices |
| :--- |
| to the number of edges? |
| 4. |
| (43\% of all $11^{\text {th }}$ graders answered this question correctly on STAAR.) |

The volume of a solid is how much it can hold or the measure of the amount of space it occupies.

It is measured in cubic units.
The formula for a cylinder is $V=B h$ or $V=\pi r^{2} h$.
The $\mathbf{B}$ stands for the area of the base and the
$\boldsymbol{h}$ stands for the height of the cylinder.


Find volume of the cylinder.


Please measure to the nearest $\frac{1}{4}$ of an inch.


1-4. Find the volume of each cylinder.

4.


Find the volume of the cylinder with radius $r$ and height $h$.
5. $r=6 \mathrm{in}, h=12 \mathrm{in}$
6. $r=2 \mathrm{~cm}, h=13 \mathrm{~cm}$
7. $r=1.9 \mathrm{~m}, h=8.7 \mathrm{~m}$

8-10. Find the volume of the solid. If two units of measure are used, give your answer in the smaller units. Round your answer to the nearest hundredth.
8. $r=6.4$ in


Find the volume of each cylinder. Round your answer to the nearest tenth if necessary. Use 3.14 for $\pi$.
1.

2.


| 3. | A cylindrical oil drum has a diameter of 2 feet and a height of 3 feet. What is <br> the volume of the oil drum? | New Oats cereal is packaged in a cardboard cylinder. The packaging is 10 <br> inches tall with a diameter of 3 inches. What is the volume of the New Oats <br> cereal package? |
| :--- | :--- | :--- |
| 5. | A small plastic storage container is in the shape of a cylinder. It has a <br> diameter of 7.6 centimeters and a height of 3 centimeters. What is the <br> volume of the storage cylinder? |  |
| 6. | A can of juice has a diameter of 6.6 centimeters and a height of 12.1 <br> centimeters. What is the total volume of a six-pack of juice cans? | Mr. Macady has an old cylindrical grain silo on his farm that stands 25 feet <br> high with a diameter of 10 feet. Mr. Macady is planning to tear down the old <br> silo and replace it with a new and bigger one. The new cylindrical silo will <br> stand 30 feet high and have a diameter of 15 feet. What is the volume of the <br> old silo? |
| 8. | In the problem above, what is the volume of the new silo? |  |
| 9. | In the problems above, how much greater is the volume of the new silo than <br> the old silo? | Matt is filling an inflatable pool with water from a hose. The pool is in the <br> shape of a cylinder and has a diameter of 7 feet. How long will it take to fill <br> the pool to a depth of 8 inches if the hose fills a 1 gallon milk jug in 3 <br> seconds? (Hint: There are 231 in ${ }^{3}$ in a gallon.) |

For the four problems below use the four corresponding pictures.


A cylindrical glass vase is 6 inches in diameter and 12 inches high. There are 3 inches of sand in the vase, as shown.

1. Which of the following is closest to the volume of the sand in the vase?
A $85 \mathrm{in}^{3}$
B $254 \mathrm{in}^{3}$
C $54 \mathrm{in}^{3}$
D $339 \mathrm{in}^{3}$

The radius of the base of a can of lemonade mix is 6 cm . The height of the
2. can is 15 cm . The lemonade mix fills the can to a height of 7 cm . What is the volume of the lemonade mix in the can?

The radius of the base of a paint can is 4 cm . The height of the can is 16 cm .
3. The paint in the can fills it to a height of 10 cm . How many liters of paint thinner must be added to the can in order to completely fill it to the top?
Note: 1 liter of paint thinner fills $1000 \mathrm{~cm}^{3}$.
4.

The radius of the base of a right circular cylinder is 8 cm . The height of the cylinder is 20 cm . Find the volume of the cylinder.

## PIECE OF CAKE VOLUME TASK

You are in charge of ordering a cake for your friend's party. Cassie's Cake Company offers 3 designs of cakes, all for the same price. Calculate which cake design gives you the greatest volume to find out which type you should buy. Show your work. Round your answers to the nearest tenth.


The volume of a cone is one third the product of the area of the base, $B$ and the height, $h$.

$$
V=\frac{1}{3} B h=\frac{1}{3} \pi r^{2} h
$$



1. A jewelry maker designs a pair of cone shaped earrings out of sterling silver. How much sterling silver is needed to make a pair of earrings?


Height of the cone $=21 \mathrm{~mm}$
Radius of the circle base $=5 \mathrm{~mm}$

Find the volume of the cone with radius $r$ and height $h$.
2. $r=8 \mathrm{in}, h=15 \mathrm{in}$
3. $r=10 \mathrm{~m}, h=9 \mathrm{~m}$
4. $r=24 \mathrm{~mm}, h=18 \mathrm{~mm}$

5-7. Find the volume of the cone. If two units of measure are used, give your answer in the smaller units. Round to the nearest tenth.


Find the volume of the cone with the given dimensions, where $r=$ radius, $d=$ diameter, and $h=$ height. If two units are used, give your answer in the smaller units. Round to the nearest tenth.
8. $r=4 \mathrm{in}, h=12 \mathrm{in}$
9. $r=2.1 \mathrm{~m}, h=84 \mathrm{~cm}$
10. $d=11 \mathrm{ft}, h=24 \mathrm{ft}$

Find the volume of each cone. Round your answer to the nearest tenth if necessary. Use 3.14 for $\pi$.
1.

2.


| 3. | The mold for a cone has a diameter of 4 inches and is 6 inches tall. What is the volume of the cone mold to the nearest tenth? |  |
| :---: | :---: | :---: |
| 4. | A medium-sized paper cone has a diameter of 8 centimeters and a height of 10 centimeters. What is the volume of the cone? |  |
| 5. | A funnel has a diameter of 9 in . and is 16 in . tall. A plug is put at the open end of the funnel. What is the volume of the cone to the nearest tenth? |  |
| 6. | A party hat has a diameter of 10 cm and is 15 cm tall. What is the volume of the hat? |  |
| 7. | Find the volume of the composite figure to the nearest tenth. <br> a. Volume of cone <br> b. Volume of cylinder <br> c. Volume of composite figure |  |
| 8. | Cone Formula: V = $\qquad$ <br> What is the height of the cone? $\qquad$ <br> What is the radius of the base? $\qquad$ |  |
| 9. | A typical lodge pole pine tree found in Montana has a trunk that is about 8 inches in diameter at its base and grows to 50 feet tall. Estimate the volume of the trunk of the tree in cubic feet. |  |
| 10. | How many lodge pole pines are needed to make 1 cord of wood? (Cord $=4$ feet by 8 feet by 4 feet. A typical cord has $20 \%$ air because of unused stacking space.) |  |

## Volume of a Sphere

$$
V=\frac{4}{3} \pi r^{3}
$$

Find the volume of each sphere. Round your answer to the nearest tenth if necessary. Use 3.14 for $\pi$. Show your work.

2.

3. $r=3$ inches
4. $d=9$ feet
5. $r=1.5$ meters

| 6. | A globe is a map of Earth shaped as a sphere. What is the volume to the <br> nearest tenth of a globe with a diameter of 16 inches? |  |
| :--- | :--- | :--- |
| 7. | The maximum diameter of a bowling ball is 8.6 inches. What is the volume <br> to the nearest tenth of a bowling ball with this diameter? |  |
| 8. | According to the National Collegiate Athletic Association men's rules, a <br> tennis ball must have a diameter of more than $2 \frac{1}{2}$ inches and less than $2 \frac{5}{8}$ <br> inches. What is the volume of a sphere with a diameter of $2 \frac{1}{2}$ inches? |  |
| 9. | In the problem above, what is the volume of a mini-tennis ball with a <br> diameter of 2 inches? |  |
| 10. | In the problems above, write an inequality that expresses the range in the <br> volume of acceptable tennis balls. |  |
| 11. | A regulation NBA basketball has a diameter of 9.4 inches. What is the <br> volume of one of these basketballs? Round to the nearest tenth. |  |

Sphere Formula: Volume = $\qquad$

Find the volume of each solid below. Round answers to the nearest tenth.
1.

2.

3.

4.

5.

6.

7. Approximately how many times as great is the volume of the grapefruit as the volume of the lime?

8. Find the volume of a sphere with a circumference of $36 \pi \mathrm{ft}$.
9. You have a glass sphere filled with water fitting exactly into a cubical box (see picture to the right), the width and height of which exactly match the diameter of the sphere. If you break the glass sphere and pour the water into the cube, how much of the cube's volume will be filled with water?


Round all answers to the nearest hundredths place unless otherwise directed.
Several students went to Baskin-Robbins to get ice cream. They found that the diameter of the scoop of ice cream was 3 inches. The cup and cone both had a diameter of 3 inches and a height of 4.5 inches.

| 1. | What is the volume of the cone? What is the volume of the <br> cup? |  |  |
| :---: | :--- | :--- | :--- |
| 2. | What is the volume of the scoop of ice cream? |  |  |
| 3. | Will either or both of the containers hold the packed ice cream? |  |  |
| 4. | What is the ratio ( $x: 1)$ of the cup's volume <br> to the scoop of ice cream's volume? <br> Explain what this means. |  |  |
| 5. | What is the ratio ( $x: 1$ ) of the cone's volume <br> to the scoop of ice cream's volume? <br> Explain what this means. |  |  |
| 6. | What is the surface area of the cup? <br> Remember the cup only has one base. |  |  |

Some students decide they want two scoops in a cone and wonder what the height of the cone would have to be to hold two scoops of packed ice cream. The diameter of the cone is still 3 inches.

| 7. | What is the volume of two scoops of ice cream? |  |
| :---: | :--- | :--- |
| 8. | If the radius of the cone remains the same, what must the <br> height of the cone be so that the two scoops of packed ice <br> cream will fill the cone without excess? Is this a reasonable <br> height for an ice cream cone? Why or why not? |  |


| 9. | The cylindrical container of ice cream has a diameter of 5 inches and a <br> height of 7.25 inches. What is its volume? |  |
| :---: | :--- | :--- |
| 10. | The product label claims that the container holds 14 scoops of ice cream <br> with each scoop having a diameter of 3 inches. Exactly how many <br> scoops of ice cream will the container really hold? |  |
| 11. | What is the surface area of the container (the container has a top)? |  |

12. A different group of students decided to go to Dairy Queen instead and get Blizzards. They saw four sizes listed with the following prices:

| The Mini - \$2.55 | The Small - \$3.25 |
| :---: | :---: |
| The Medium - \$3.80 | The Large - \$4.65 |


| Cylindrical cup sizes |  |
| :---: | :---: |
| The Mini -7.2 cm high and 5.5 cm diameter | The Small -8.3 cm high and 6.3 cm diameter |
| The Medium -11 cm high and 6.5 cm <br> diameter | The Large -16 cm high and 6 cm diameter |

If they wanted to get the most Blizzard for their money, then which size should they buy? Complete the table below to find out. Since you are finding the surface area of the ice cream, use 2 bases.

|  | Surface Area <br> of Blizzard <br> (hundredth) | Volume of <br> Blizzard <br> (hundredth) | Cost | \$ per cm <br> (ten-thousandth) | $\mathbf{c m}^{\mathbf{3}}$ per \$ <br> (ten-thousandth) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Mini |  |  |  |  |  |
| Small |  |  |  |  |  |
| Medium |  |  |  |  |  |
| Large |  |  |  |  |  |


| $*$ | What is the best flavor of Blizzard? If you have never had a <br> Blizzard, check DairQueen.com and determine which one <br> you think would be the best? |  |
| :--- | :--- | :--- |

https://www.youtube.com/watch?v=yJZ5RJxfRIc

## Surface Area

## Prism

Cylinder

## Lateral

$$
S=P h
$$

$S=2 \pi r h$
$S=2 \pi r h+2 \pi r^{2}$

## Total

$S=P h+2 B$

Find the surface area of each prism.
1.
2.
3.

2.

5. Marita is decorating the prism at the right with tiles. Each tile is 1 square foot. Each tile costs $\$ 0.45$. How much will it cost Marita to tile the whole prism?


Find lateral surface area and total surface area of the cylinders.

6.
7.


The surface area of a polyhedron is the sum of the areas of its faces. The surface area of a cylinder is the sum of twice the area of the base and the product of the base's circumference and the height.


1. Find the surface area of a stack of CDs.

2. Find the surface area of a cylinder that has a radius of 5 feet and a height of 8 feet.

Sketch a cylinder with radius $r$ and height $h$. Then find its surface area. Use 3.14 for pi.
3. $r=4 \mathrm{~cm}, h=8 \mathrm{~cm}$
4. $r=10 \mathrm{~cm}, h=12 \mathrm{~cm}$
5. $r=3 \mathrm{ft}, h=21 \mathrm{ft}$
$6-8$. For the three shapes below, find the surface area. Use 3.14 for pi.

9. A factory specializes in producing plastic, cylindrical tennis ball cans. It can produce 2000 cans per day. Each can has a diameter of $4 \frac{2}{5}$ inches and a height of 11 inches. How many square inches of plastic does the factory use per day to make the cans? Assume plastic is used for all the lateral area plus one base.
10. If the factory also produces paper labels for the cans, what is the area of one label?

## Solve the following application problems. Draw a picture to help you.

| 1. | Campbell's soup company is having a contest for students at DIS to redesign <br> the label for the chicken noodle soup. If the diameter of the can is 3 in, and the <br> height is 4 in, how much paper do students need to create their design? |
| :--- | :--- |
| 2.Susan has a fish tank in the shape of a cylinder that is 26 inches tall. The <br> diameter of the tank is 12 inches. If there are 2 inches of rocks in the bottom, <br> how much water is needed to fill the tank? |  |
| 3. $\mathrm{V}=$ |  |

Find the surface area of each figure. Don't forget to include units!
5.

6.

\#7-9: Find the lateral surface area of each cylinder. \#10-12 Find the total surface area of each cylinder.
Round your answers to the nearest tenth, if necessary. Use 3.14 for $\pi$.
7.

8.

9.

10.

11.

12.


## The Buffalo snowstorm has buried the Bills' Ralph Wilson Stadium



In November 2014 Buffalo, NY was hit with multiple snowstorms that ended up dumping about 5 feet of snow on Ralph Wilson Stadium. The Buffalo Bills were to play the New York Jets that following Sunday and the ground crew had no idea if they would be able to get the stadium ready in time for the game. Here are some headlines about the event:

The Bills estimate they will need to remove 220,000 tons of snow to clear the stadium for Sunday's game. The team is seeking at least 500 fans -- working on three shifts -- to shovel out the stadium. In return, the Bills will pay fans $\$ 10$ per hour and offer free game tickets.

Assuming one person was shoveling at a rate of two scoops per minute, it'd take about 33.5 years to entirely clear the stadium.

If 500 people show up to shovel, it'd take them each about 35,200 scoops to clear the whole stadium. But at that rate, it'll still take three and a half weeks to clear the stadium.

Let's use some math to determine if the statements above make sense.

Conversions we need to know:

| 1 acre $=4840$ square yards | 1 square yard $=9$ square feet |
| :---: | :---: |
| $A=\pi r^{2}$ | 1 cubic yard $=27$ cubic feet |
| $V=\pi r^{2} h($ cylinder $)$ | $V=l w h \quad$ (rectangular prism) |

You may use a calculator for all math involved with this project.

|  | How big is Ralph Wilson Stadium? <br> Ralph Wilson Stadium is an oval. However, you can estimate the area of <br> the stadium using a circle with a radius of 365 feet. What is the area inside <br> the stadium in square feet? In square yards? In acres? |  |
| :--- | :--- | :--- |
|  | How much is one ton of snow? <br> One ton of snow can vary quite a bit depending on how wet or dry the <br> snow is. For our purposes we will estimate one ton on snow to be a block <br> of snow 10 ft. by 20 ft. by 1 ft. deep. What is the volume of one ton of <br> snow? |  |
| 3. | What is the volume of the snow that fell? <br> We need to determine the entire volume of snow inside the stadium. They <br> estimated that 5 feet of snow fell. What is the volume of snow inside the <br> stadium in cubic feet? In cubic yards? |  |
| 4. | In tons, how much does all the snow inside the stadium weigh? |  |

